PROBLEM 7-6a

Statement: The link lengths and the values of θ_2, ω_2, and α_2 for the some non inverted offset fourbar slider-crank linkages are defined in Table P7-2. The general linkage configuration and terminology are shown in Figure P7-2. For row *a*, draw the linkage to scale and find the accelerations of the pin joints *A* and *B* the acceleration of slip at the sliding joint using an analytical method.

Given: Link lengths:

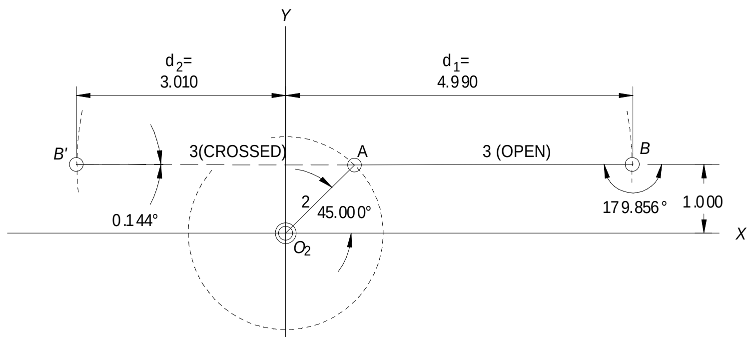
Link 2 $a := 1.4 \cdot in$ Link 3 $b := 4 \cdot in$

Offset: $c := 1 \cdot in$

Link 2 position, velocity, and acceleration: $\theta_2 := 45 \cdot deg$ $\omega_2 := 10 \cdot \frac{rad}{sec}$ $\alpha_2 := 0 \cdot \frac{rad}{sec^2}$

Solution: See Figure P7-2 and Mathcad file P0706a.

1. Draw the linkage to scale and label it.



2. Determine θ_3 and *d* using equations 4.16 and 4.17.

Open:

$$\theta_{32} := asin\left(-\frac{a \cdot sin(\theta_2) - c}{b}\right) + \pi \qquad \theta_{32} = 180.144\, deg$$

$$d_2 := a \cdot cos(\theta_2) - b \cdot cos(\theta_{32}) \qquad d_2 = 4.990\, in$$

Crossed:

$$\theta_{31} := asin\left(\frac{a \cdot sin(\theta_2) - c}{b}\right) \qquad \theta_{31} = -0.144\, deg$$

$$d_1 := a \cdot cos(\theta_2) - b \cdot cos(\theta_{31}) \qquad d_1 = -3.010\, in$$

3. Determine the angular velocity of link 3 using equation 6.22a.

Open

$$\omega_{32} := \frac{a}{b} \cdot \frac{cos(\theta_2)}{cos(\theta_{32})} \cdot \omega_2 \qquad \omega_{32} = -2.475\, \frac{rad}{sec}$$

Crossed

$$\omega_{31} := \frac{a}{b} \cdot \frac{cos(\theta_2)}{cos(\theta_{31})} \cdot \omega_2 \qquad \omega_{31} = 2.475\, \frac{rad}{sec}$$

4. Using the Euler identity to expand equation 7.15b for $\mathbf{A}_A$, determine its magnitude, and direction.

$$\mathbf{A_A} := a\cdot\alpha_2\cdot\left(-sin\left(\theta_2\right) + j\cdot cos\left(\theta_2\right)\right) - a\cdot\omega_2^{\,2}\cdot\left(cos\left(\theta_2\right) + j\cdot sin\left(\theta_2\right)\right)$$

$$\mathbf{A_A} = -98.995 - 98.995j\,\frac{in}{sec^2} \qquad A_A := \left|\mathbf{A_A}\right| \qquad \theta_{AA} := arg\left(\mathbf{A_A}\right)$$

The acceleration of pin A is $A_A = 140.0\,\frac{in}{sec^2}$ at $\theta_{AA} = -135.0\,deg$

5. Determine the angular acceleration of link 3 using equation 7.16d.

Open
$$\alpha_{32} := \frac{a\cdot\alpha_2\cdot cos\left(\theta_2\right) - a\cdot\omega_2^{\,2}\cdot sin\left(\theta_2\right) + b\cdot\omega_{32}^{\,2}\cdot sin\left(\theta_{32}\right)}{b\cdot cos\left(\theta_{32}\right)}$$

$$\alpha_{32} = 24.764\,\frac{rad}{sec^2}$$

Crossed
$$\alpha_{31} := \frac{a\cdot\alpha_2\cdot cos\left(\theta_2\right) - a\cdot\omega_2^{\,2}\cdot sin\left(\theta_2\right) + b\cdot\omega_{31}^{\,2}\cdot sin\left(\theta_{31}\right)}{b\cdot cos\left(\theta_{31}\right)}$$

$$\alpha_{31} = -24.764\,\frac{rad}{sec^2}$$

6. Use equation 7.16e for the acceleration of pin B.

Open:

$$A_{B2} := -a\cdot\alpha_2\cdot sin\left(\theta_2\right) - a\cdot\omega_2^{\,2}\cdot cos\left(\theta_2\right) + b\cdot\alpha_{32}\cdot sin\left(\theta_{32}\right) + b\cdot\omega_{32}^{\,2}\cdot cos\left(\theta_{32}\right)$$

$$A_{B2} = -123.7\,\frac{in}{sec^2}$$

A negative sign means that $\mathbf{A_B}$ is to the left

Crossed:

$$A_{B1} := -a\cdot\alpha_2\cdot sin\left(\theta_2\right) - a\cdot\omega_2^{\,2}\cdot cos\left(\theta_2\right) + b\cdot\alpha_{31}\cdot sin\left(\theta_{31}\right) + b\cdot\omega_{31}^{\,2}\cdot cos\left(\theta_{31}\right)$$

$$A_{B1} = -74.2\,\frac{in}{sec^2}$$

A negative sign means that $\mathbf{A_B}$ is to the left

PROBLEM 7-7a

Statement: The link lengths and the values of θ_2, ω_2, and γ for an inverted fourbar slider-crank linkage are defined in Table P7-3, row *a*, and are given below. Find the acceleration of the pin joints *A* and *B* and the acceleration of slip at the sliding joint. Solve by the analytic vector loop method of Section 7.3 for the open configuration of the linkage.

Given: Link lengths:

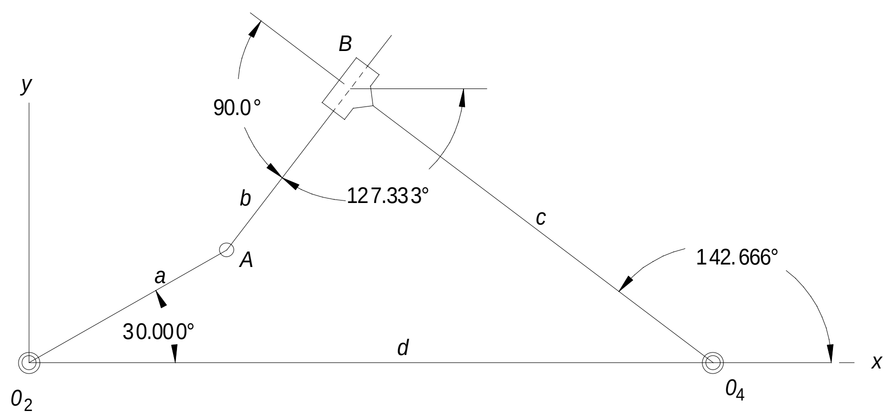
Link 2 $a := 2 \cdot in$ Link 4 $c := 4 \cdot in$ Link 1 $d := 6 \cdot in$

Angle between links 3 and 4 $\gamma := 90 \cdot deg$

Link 2 position, velocity, and accel. $\theta_2 := 30 \cdot deg$ $\omega_2 := 10 \cdot \frac{rad}{sec}$ $\alpha_2 := -25 \cdot \frac{rad}{sec^2}$

Solution: See Figure P7-3 and Mathcad file P0707a.

1. Draw the linkage to scale and label it.



2. Use the equations in Section 4.7 to solve for the positions of links 3 and 4 and for the length b.

$P := a \cdot sin(\theta_2) \cdot sin(\gamma) + (a \cdot cos(\theta_2) - d) \cdot cos(\gamma)$ $P = 1.000\ in$

$Q := -a \cdot sin(\theta_2) \cdot cos(\gamma) + (a \cdot cos(\theta_2) - d) \cdot sin(\gamma)$ $Q = -4.268\ in$

$R := -c \cdot sin(\gamma)$ $S := R - Q$ $T := 2 \cdot P$ $U := Q + R$

$R = -4.000\ in$ $S = 0.268\ in$ $T = 2.000\ in$ $U = -8.268\ in$

$\theta_4 := 2 \cdot atan2\left[(2 \cdot S), \left(-T + \sqrt{T^2 - 4 \cdot S \cdot U}\right)\right]$ $\theta_4 = 142.667\ deg$

$\theta_3 := \theta_4 + \gamma$ $\theta_3 = 232.667\ deg$

$b := \frac{a \cdot sin(\theta_2) - c \cdot sin(\theta_4)}{sin(\theta_3)}$ $b = 1.793\ in$

3. Calculate the angular velocity of links 3 and 4 and the slip velocity using equations 6.30.

$$\omega_4 := \frac{a\cdot\omega_2\cdot cos(\theta_2 - \theta_3)}{b + c\cdot cos(\gamma)} \qquad \omega_4 = -10.292\,\frac{rad}{sec}$$

$$bdot := \frac{-a\cdot\omega_2\cdot sin(\theta_2) + \omega_4\cdot(b\cdot sin(\theta_3) + c\cdot sin(\theta_4))}{cos(\theta_3)} \qquad bdot = 33.461\,\frac{in}{sec}$$

$$\omega_3 := \omega_4$$

4. Solve for the accelerations using equations (7.26) and (7.27).

$$P := a\cdot\alpha_2\cdot cos(\theta_3 - \theta_2) \qquad P = 46.138\,in\cdot sec^{-2}$$

$$Q := a\cdot\omega_2^{\,2}\cdot sin(\theta_3 - \theta_2) \qquad Q = -77.075\,in\cdot sec^{-2}$$

$$R := c\cdot\omega_4^{\,2}\cdot sin(\theta_4 - \theta_3) \qquad R = -423.705\,in\cdot sec^{-2}$$

$$S := 2\cdot bdot\cdot\omega_3 \qquad S = -688.757\,in\cdot sec^{-2}$$

$$T := b + c\cdot cos(\theta_3 - \theta_4) \qquad T = 1.793\,in$$

$$\alpha_4 := \frac{P + Q + R - S}{T} \qquad \alpha_4 = 130.56\,rad\cdot sec^{-2}$$

$$K := a\cdot\omega_2^{\,2}\cdot(b\cdot cos(\theta_3 - \theta_2) + c\cdot cos(\theta_4 - \theta_2)) \qquad K = -639.230\,in^2\cdot sec^{-2}$$

$$L := a\cdot\alpha_2\cdot(b\cdot sin(\theta_2 - \theta_3) - c\cdot sin(\theta_4 - \theta_2)) \qquad L = 150.000\,in^2\cdot sec^{-2}$$

$$M := -\omega_4^{\,2}\cdot\left(b^2 + c^2 + 2\cdot b\cdot c\cdot cos(\theta_4 - \theta_3)\right) \qquad M = -2.035\times 10^3\,in^2\cdot sec^{-2}$$

$$N := 2\cdot bdot\cdot c\cdot\omega_4\cdot sin(\theta_4 - \theta_3) \qquad N = 2.755\times 10^3\,in^2\cdot sec^{-2}$$

$$bddot := -\frac{K + L + M + N}{T} \qquad bddot = -128.48\,\frac{in}{sec^2}$$

The acceleration of slip is *bddot.* It is directed along link 3, positive inward from *B* towards *A*, so that its angle is θ_3. For the acceleration of the pin joints *A* and *B*,

$$\mathbf{A_A} := a\cdot\alpha_2\cdot(-sin(\theta_2) + j\cdot cos(\theta_2)) - a\cdot\omega_2^{\,2}\cdot(cos(\theta_2) + j\cdot sin(\theta_2))$$

$$A_A := |\mathbf{A_A}| \qquad A_A = 206.155\,\frac{in}{sec^2} \qquad \theta_{AA} := arg(\mathbf{A_A}) \qquad \theta_{AA} = -135.964\,deg$$

$$\mathbf{A_B} := -c\cdot\alpha_4\cdot(sin(\theta_4) - j\cdot cos(\theta_4)) - c\cdot\omega_4^{\,2}\cdot(cos(\theta_4) + j\cdot sin(\theta_4))$$

$$A_B := |\mathbf{A_B}| \qquad A_B = 672.505\,\frac{in}{sec^2} \qquad \theta_{AB} := arg(\mathbf{A_B}) \qquad \theta_{AB} = -88.280\,deg$$

PROBLEM 7-8a

Statement: The link lengths and the values of θ_2, ω_2, and γ for an inverted fourbar slider-crank linkage are defined in Table P7-3, row *a*, and are given below. Find the acceleration of the pin joints *A* and *B* and the acceleration of slip at the sliding joint. Solve by the analytic vector loop method of Section 7.3 for the crossed configuration of the linkage.

Given: Link lengths:

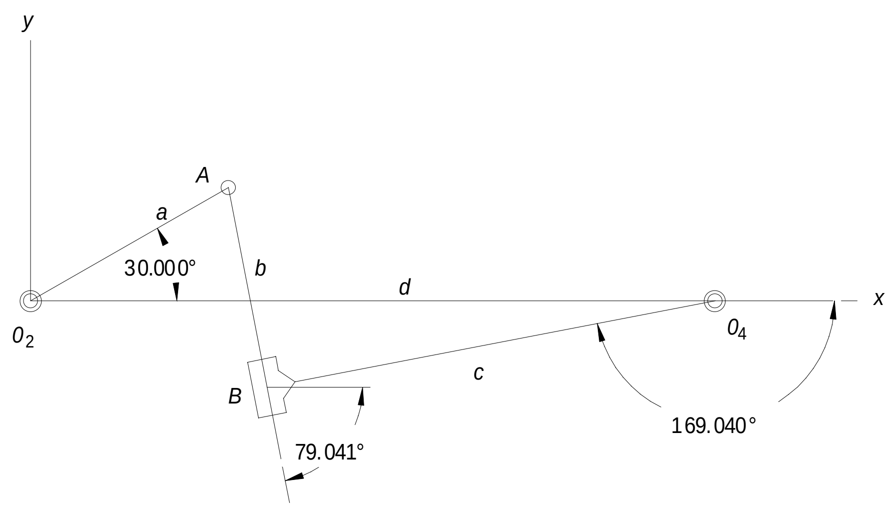
Link 2 $a := 2 \cdot in$ Link 4 $c := 4 \cdot in$ Link 1 $d := 6 \cdot in$

Angle between links 3 and 4 $\gamma := 90 \cdot deg$

Link 2 position, velocity, and accel. $\theta_2 := 30 \cdot deg$ $\omega_2 := 10 \cdot \frac{rad}{sec}$ $\alpha_2 := -25 \cdot \frac{rad}{sec^2}$

Solution: See Figure P7-3 and Mathcad file P0708a.

1. Draw the linkage to scale and label it.



2. Use the equations in Section 4.7 to solve for the positions of links 3 and 4 and for the length b.

$$P := a \cdot sin(\theta_2) \cdot sin(\gamma) + (a \cdot cos(\theta_2) - d) \cdot cos(\gamma) \qquad P = 1.000\, in$$

$$Q := -a \cdot sin(\theta_2) \cdot cos(\gamma) + (a \cdot cos(\theta_2) - d) \cdot sin(\gamma) \qquad Q = -4.268\, in$$

$$R := -c \cdot sin(\gamma) \qquad S := R - Q \qquad T := 2 \cdot P \qquad U := Q + R$$

$$R = -4.000\, in \qquad S = 0.268\, in \qquad T = 2.000\, in \qquad U = -8.268\, in$$

$$\theta_4 := 2 \cdot atan2\left[(2 \cdot S), -T - \sqrt{T^2 - 4 \cdot S \cdot U} \right] \qquad \theta_4 = -169.041\, deg$$

$$\theta_3 := \theta_4 - \gamma \qquad \theta_3 = -259.041\, deg$$

$$b := \frac{a \cdot sin(\theta_2) - c \cdot sin(\theta_4)}{sin(\theta_3)} \qquad b = 1.793\, in$$

3. Calculate the angular velocity of links 3 and 4 and the slip velocity using equations 6.30.

$$\omega_4 := \frac{a \cdot \omega_2 \cdot cos(\theta_2 - \theta_3)}{b + c \cdot cos(\gamma)} \qquad \omega_4 = 3.639 \frac{rad}{sec}$$

$$bdot := \frac{-a \cdot \omega_2 \cdot sin(\theta_2) + \omega_4 \cdot (b \cdot sin(\theta_3) + c \cdot sin(\theta_4))}{cos(\theta_3)} \qquad bdot = 33.461 \frac{in}{sec}$$

$$\omega_3 := \omega_4$$

4. Solve for the accelerations using equations (7.26) and (7.27).

$$P := a \cdot \alpha_2 \cdot cos(\theta_3 - \theta_2) \qquad P = -16.312\, in \cdot sec^{-2}$$

$$Q := a \cdot \omega_2^{\,2} \cdot sin(\theta_3 - \theta_2) \qquad Q = 189.057\, in \cdot sec^{-2}$$

$$R := c \cdot \omega_4^{\,2} \cdot sin(\theta_4 - \theta_3) \qquad R = 52.961\, in \cdot sec^{-2}$$

$$S := 2 \cdot bdot \cdot \omega_3 \qquad S = 243.508\, in \cdot sec^{-2}$$

$$T := b + c \cdot cos(\theta_3 - \theta_4) \qquad T = 1.793\, in$$

$$\alpha_4 := \frac{P + Q + R - S}{T} \qquad \alpha_4 = -9.93\, rad \cdot sec^{-2}$$

$$K := a \cdot \omega_2^{\,2} \cdot (b \cdot cos(\theta_3 - \theta_2) + c \cdot cos(\theta_4 - \theta_2)) \qquad K = -639.230\, in^2 \cdot sec^{-2}$$

$$L := a \cdot \alpha_2 \cdot (b \cdot sin(\theta_2 - \theta_3) - c \cdot sin(\theta_4 + \theta_2)) \qquad L = -46.352\, in^2 \cdot sec^{-2}$$

$$M := -\omega_4^{\,2} \cdot \left(b^2 + c^2 + 2 \cdot b \cdot c \cdot cos(\theta_4 - \theta_3)\right) \qquad M = -254.418\, in^2 \cdot sec^{-2}$$

$$N := 2 \cdot bdot \cdot c \cdot \omega_4 \cdot sin(\theta_4 - \theta_3) \qquad N = 974.033\, in^2 \cdot sec^{-2}$$

$$bddot := -\frac{K + L + M + N}{T} \qquad bddot = -18.98 \frac{in}{sec^2}$$

The acceleration of slip is bddot. It is directed along link 3, positive inward from *B* towards *A*, so that its angle is θ_3. For the acceleration of the pin joints *A* and *B*,

$$\mathbf{A_A} := a \cdot \alpha_2 \cdot (-sin(\theta_2) + j \cdot cos(\theta_2)) - a \cdot \omega_2^{\,2} \cdot (cos(\theta_2) + j \cdot sin(\theta_2))$$

$$A_A := |\mathbf{A_A}| \qquad A_A = 206.155 \frac{in}{sec^2} \qquad \theta_{AA} := arg(\mathbf{A_A}) \qquad \theta_{AA} = -135.964\, deg$$

$$\mathbf{A_B} := -c \cdot \alpha_4 \cdot (sin(\theta_4) - j \cdot cos(\theta_4)) - c \cdot \omega_4^{\,2} \cdot (cos(\theta_4) + j \cdot sin(\theta_4))$$

$$A_B := |\mathbf{A_B}| \qquad A_B = 66.195 \frac{in}{sec^2} \qquad \theta_{AB} := arg(\mathbf{A_B}) \qquad \theta_{AB} = 47.822\, deg$$

PROBLEM 7-12

Statement: You are riding on a carousel that is rotating at a constant 15 rpm. It has an inside radius of 3 ft and an outside radius of 10 ft. You begin to run from the inside to the outside along a radius. Your peak velocity with respect to the carousel is 5 mph and occurs at a radius of 7 ft. What is your maximum Coriolis acceleration magnitude and its direction with respect to the carousel?

Units: $rpm := 2\cdot\pi\cdot rad\cdot min^{-1}$

Given:

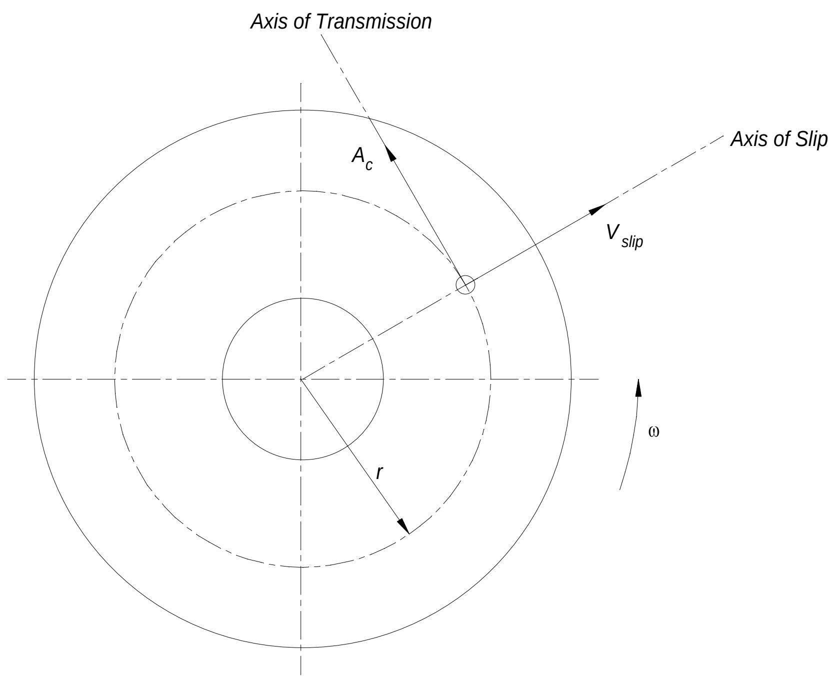
Carousel angular velocity $\omega := 15\cdot rpm$ $\qquad \omega = 1.571\,\frac{rad}{sec}$

Peak velocity $V_{slip} := 5\cdot mph$ $\qquad V_{slip} = 7.333\,\frac{ft}{sec}$

Radius at peak velocity $r := 7\cdot ft$

Solution: See Mathcad file P0712.

1. Draw a plan view of the carousel floor showing your position at peak velocity and the velocity and Coriolis acceleration vectors.



2. The direction of your path defines the axis of slip. The transmission axis is perpendicular to the axis of slip and positive in the direction of the tangential velocity of the carousel. Thus, the direction of the Coriolis acceleration vector is along the positive transmission axis.

3. Use equation 7.19 to calculate the magnitude of the Coriolis component of your acceleration.

$$A_c := 2\cdot V_{slip}\cdot\omega \qquad A_c = 23.038\,\frac{ft}{sec^2} \qquad A_c = 276.46\,\frac{in}{sec^2}$$

PROBLEM 7-13b

Statement: The linkage in Figure P7-5a has the dimensions and crank angle given below. Find α_3, $\mathbf{A}_A$, $\mathbf{A}_B$, and $\mathbf{A}_C$ for the position shown for ω_2 = 15 rad/sec and α_2 = 10 rad/sec^2 in the direction shown. Use an analytical method.

Given: Link lengths:

Link 2 $a := 0.8 \cdot in$ Link 3 $b := 1.93 \cdot in$

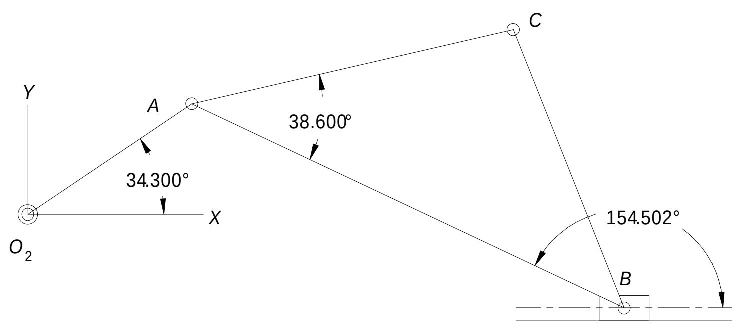
Offset: $c := -0.38 \cdot in$

Coupler point data: $p := 1.33 \cdot in$ $\delta_3 := 38.6 \cdot deg$

Link 2 position, velocity, and acceleration: $\theta_2 := 34.3 \cdot deg$ $\omega_2 := 15 \cdot \frac{rad}{sec}$ $\alpha_2 := 10 \cdot \frac{rad}{sec^2}$

Solution: See Figure P7-5a and Mathcad file P0713b.

1. Draw the linkage to scale and label it.



2. Determine θ_3 and d using equations 4.16 and 4.17.

$$\theta_3 := asin\left(-\frac{a \cdot sin(\theta_2) - c}{b}\right) + \pi \qquad \theta_3 = 154.502\, deg$$

$$d := a \cdot cos(\theta_2) - b \cdot cos(\theta_3) \qquad d = 2.403\, in$$

3. Determine the angular velocity of link 3 using equation 6.22a.

$$\omega_3 := \frac{a}{b} \cdot \frac{cos(\theta_2)}{cos(\theta_3)} \cdot \omega_2 \qquad \omega_3 = -5.691\, \frac{rad}{sec}$$

4. Using the Euler identity to expand equation 7.15b for $\mathbf{A}_A$, determine its magnitude, and direction.

$$\mathbf{A_A} := a \cdot \alpha_2 \cdot \left(-sin(\theta_2) + j \cdot cos(\theta_2)\right) - a \cdot \omega_2^{\,2} \cdot \left(cos(\theta_2) + j \cdot sin(\theta_2)\right)$$

$$\mathbf{A_A} = -153.206 - 94.826j\, \frac{in}{sec^2} \qquad A_A := \left|\mathbf{A_A}\right| \qquad \theta_{AA} := arg(\mathbf{A_A})$$

The acceleration of pin A is $A_A = 180\, \frac{in}{sec^2}$ at $\theta_{AA} = -148.2\, deg$

5. Determine the angular acceleration of link 3 using equation 7.16d.

$$\alpha_3 := \frac{a\cdot\alpha_2\cdot cos(\theta_2) - a\cdot\omega_2{}^2\cdot sin(\theta_2) + b\cdot\omega_3{}^2\cdot sin(\theta_3)}{b\cdot cos(\theta_3)} \qquad \alpha_3 = 38.990\,\frac{rad}{sec^2}$$

6. Use equation 7.16e for the acceleration of pin B.

$$A_B := -a\cdot\alpha_2\cdot sin(\theta_2) - a\cdot\omega_2{}^2\cdot cos(\theta_2) + b\cdot\alpha_3\cdot sin(\theta_3) + b\cdot\omega_3{}^2\cdot cos(\theta_3)$$

$A_B = -177.2\,\frac{in}{sec^2}$ A negative sign means that $\mathbf{A_B}$ is to the left

7. Determine the acceleration of the coupler point C using equations 7.32. Note that θ_3 is defined from point B in Figure 7-6 and from point A in Figure 7-9. To use equation 7.32 for a slider-crank we must redefine θ_3.

$\theta_3 := \theta_3 - 180\cdot deg \qquad \theta_3 = -25.498\,deg$

$$\mathbf{A_{CA}} := p\cdot\alpha_3\cdot\left(-sin(\theta_3 + \delta_3) + j\cdot cos(\theta_3 + \delta_3)\right) \ldots$$
$$+ -p\cdot\omega_3{}^2\cdot\left(cos(\theta_3 + \delta_3) + j\cdot sin(\theta_3 + \delta_3)\right)$$

$$\mathbf{A_C} := \mathbf{A_A} + \mathbf{A_{CA}}$$

$\mathbf{A_C} = -206.910 - 54.083j\,\frac{in}{sec^2} \qquad A_C := \left|\mathbf{A_C}\right| \qquad \theta_{AC} := arg(\mathbf{A_C})$

The acceleration of point C is $A_C = 213.861\,\frac{in}{sec^2}$ at $\theta_{AC} = -165.352\,deg$

PROBLEM 7-15b

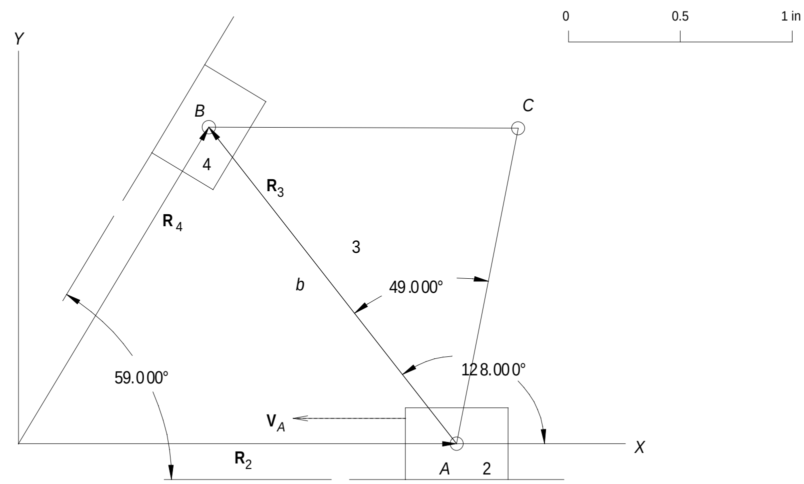
Statement: The linkage in Figure P7-5c has the dimensions and coupler angle given below. Find α_3, $\mathbf{A}_B$, and $\mathbf{A}_C$ for the position shown for $\mathbf{V}_A$ = 10 in/sec and $\mathbf{A}_A$ = 15 in/sec^2 in the directions shown. Use an analytical method.

Given: Link lengths and angles:

Link 3 (*A* to *B*)	$b := 1.8 \cdot in$
Coupler angle	$\theta_3 := 128 \cdot deg$
Slider 4 angle	$\theta_4 := 59 \cdot deg$
Coupler point:	
Distance *A* to *C*	$R_{ca} := 1.44 \cdot in$
Angle *BAC*	$\delta_3 := -49 \cdot deg$
Input slider motion	$V_A := -10 \cdot in \cdot sec^{-1}$ $\quad A_A := 15 \cdot in \cdot sec^{-2}$

Solution: See Figure P7-5c and Mathcad file P0715b.

1. Draw the mechanism to scale and define a vector loop using the fourbar slider-crank derivation in Section 7.3 as a model.



2. Write the vector loop equation, differentiate it, expand the result and separate into real and imaginary parts to solve for ω_3 and $\mathbf{V}_B$.

$$\mathbf{R_2} + \mathbf{R_3} := \mathbf{R_4} \qquad a \cdot e^{j \cdot \theta_2} + b \cdot e^{j \cdot \theta_3} := c \cdot e^{j \cdot \theta_4}$$

where *a* is the distance from the origin to point *A*, a variable; *b* is the distance from *A* to *B*, a constant; and *c* is the distance from the origin to point *B*, a variable. Angle θ_2 is zero, θ_3 is the angle that *AB* makes with the *x* axis, and θ_4 is the constant angle that slider 4 makes with the *x* axis. Differentiating,

$$\frac{d}{dt}a + j \cdot b \cdot \omega_3 \cdot e^{j \cdot \theta_3} := \left(\frac{d}{dt}c\right) \cdot e^{j \cdot \theta_4}$$

Substituting the Euler equivalents,

$$\frac{d}{dt}a + b \cdot \omega_3 \cdot \left(-sin\left(\theta_3\right) + j \cdot cos\left(\theta_3\right)\right) := \left(\frac{d}{dt}c\right) \cdot \left(cos\left(\theta_4\right) + j \cdot sin\left(\theta_4\right)\right)$$

Separating into real and imaginary components and solving for ω_3. Note that $dc/dt = V_B$ and $da/dt = V_A$

$$\omega_3 := \frac{V_A \cdot tan(\theta_4)}{b \cdot (sin(\theta_3) \cdot tan(\theta_4) + cos(\theta_3))} \qquad \omega_3 = -13.288 \frac{rad}{sec}$$

3. Differentiate the velocity equation, expand it and solve for α_3 and A_B.

$$\frac{d^2}{dt^2}\left(a + b \cdot \alpha_3 \cdot j \cdot e^{j \cdot \theta_3} + b \cdot \omega_3^2 \cdot j \cdot e^{j \cdot \theta_3}\right) := \frac{d^2}{dt^2} c \cdot e^{j \cdot \theta_4}$$

Substituting the Euler equivalents,

$$\frac{d^2}{dt^2}a + b \cdot \alpha_3 (-sin(\theta_3) + j \cdot cos(\theta_3)) \ldots := 0$$

$$+ -b \cdot \omega_3^2 \cdot (cos(\theta_3) + j \cdot sin(\theta_3)) - \frac{d^2}{dt^2} c \cdot (cos(\theta_4) + j \cdot sin(\theta_4))$$

Separating into real and imaginary components and solving for α_3 and A_B. Note that $d^2c/dt^2 = A_B$ and $d^2a/dt^2 = A_A$

$$\alpha_3 := \frac{A_A \cdot sin(\theta_4) - b \cdot \omega_3^2 \cdot sin(\theta_4 - \theta_3)}{b \cdot cos(\theta_4 - \theta_3)} \qquad \alpha_3 = 479.924 \frac{rad}{sec^2}$$

$$A_B := \frac{b \cdot \alpha_3 \cdot cos(\theta_3) - b \cdot \omega_3^2 \cdot sin(\theta_3)}{sin(\theta_4)} \qquad A_B = -912.662 \frac{in}{sec^2}$$

4. Determine the acceleration of the coupler point C using equations 7.32.

$$\mathbf{A_{CA}} := R_{ca} \cdot \alpha_3 \cdot (-sin(\theta_3 + \delta_3) + j \cdot cos(\theta_3 + \delta_3)) - R_{ca} \cdot \omega_3^2 \cdot (cos(\theta_3 + \delta_3) + j \cdot sin(\theta_3 + \delta_3))$$

$$\mathbf{A_{CA}} = -726.910 - 117.729j \frac{in}{sec^2}$$

$$\mathbf{A_A} := A_A \qquad \mathbf{A_C} := \mathbf{A_A} + \mathbf{A_{CA}}$$

$$\mathbf{A_C} = -711.910 - 117.729j \frac{in}{sec^2} \qquad |\mathbf{A_C}| = 721.579 \frac{in}{sec^2} \qquad arg(\mathbf{A_C}) = -170.610\, deg$$

PROBLEM 7-22

Statement: The linkage in Figure P7-8a has the dimensions and crank angle given below. Find α_4, $\mathbf{A}_A$, and $\mathbf{A}_B$ in the global coordinate system for the position shown for ω_2 = 15 rad/sec clockwise (CW) and α_2 = 25 rad/sec^2 CCW. Use an analytical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 116 \cdot mm$	Link 3 (A to B)	$b := 108 \cdot mm$
Link 4 (B to O_4)	$c := 110 \cdot mm$	Link 1 (O_2 to O_4)	$d := 174 \cdot mm$

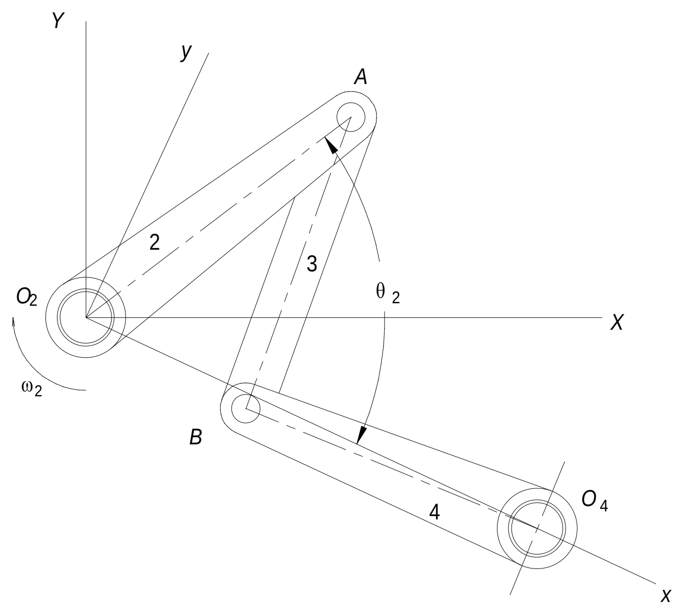
Crank angle: $\theta_2 := 62 \cdot deg$ Local xy system

Input crank angular velocity $\omega_2 := -15 \cdot rad \cdot sec^{-1}$ $\alpha_2 := 25 \cdot rad \cdot sec^{-2}$

Coordinate rotation angle $\beta := -25 \cdot deg$ Global XY system to local xy system

Solution: See Figure P7-8a and Mathcad file P0722.

1. Draw the linkage to scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$K_1 := \dfrac{d}{a}$ $K_2 := \dfrac{d}{c}$

$K_1 = 1.5000$ $K_2 = 1.5818$

$K_3 := \dfrac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$ $K_3 = 1.7307$

$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$

$B := -2 \cdot \sin(\theta_2)$

$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$

$A = -0.0424$ $B = -1.7659$ $C = 2.0186$

3. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_4 := 2 \cdot \left(atan2\left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C}\right)\right) \qquad \theta_4 = 182.681\ deg$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$K_4 := \dfrac{d}{b}$ $K_5 := \dfrac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b}$ $K_4 = 1.6111$ $K_5 = -1.7280$

$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$ $D = -2.0021$

$E := -2 \cdot \sin(\theta_2)$ $E = -1.7659$

$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$ $F = 0.0589$

5. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3 := 2\cdot\left(atan2\left(2\cdot D, -E + \sqrt{E^2 - 4\cdot D\cdot F}\right)\right) \qquad \theta_3 = 275.133\,deg$$

6. Determine the angular velocity of links 3 and 4 for the crossed circuit using equations 6.18.

$$\omega_3 := \frac{a\cdot\omega_2}{b}\cdot\frac{sin(\theta_4 - \theta_2)}{sin(\theta_3 - \theta_4)} \qquad \omega_3 = -13.869\,\frac{rad}{sec}$$

$$\omega_4 := \frac{a\cdot\omega_2}{c}\cdot\frac{sin(\theta_2 - \theta_3)}{sin(\theta_4 - \theta_3)} \qquad \omega_4 = 8.654\,\frac{rad}{sec}$$

7. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_{A}$. Determine the magnitude, and direction (in the global coordinate system).

$$\mathbf{A_A} := a\cdot\alpha_2\cdot\left(-sin(\theta_2) + j\cdot cos(\theta_2)\right) - a\cdot\omega_2^{\,2}\cdot\left(cos(\theta_2) + j\cdot sin(\theta_2)\right)$$

$$\mathbf{A_A} = -14814 - 21683j\,\frac{mm}{sec^2} \qquad A_A := \left|\mathbf{A_A}\right| \qquad \theta_{AA} := arg(\mathbf{A_A}) + \beta$$

The acceleration of pin A is $A_A = 26261\,\frac{mm}{sec^2}$ at $\theta_{AA} = -149.3\,deg$

8. Use equations 7.12 to determine the angular accelerations of links 3 and 4 for the crossed circuit.

$$A := c\cdot sin(\theta_4) \qquad B := b\cdot sin(\theta_3) \qquad D := c\cdot cos(\theta_4) \qquad E := b\cdot cos(\theta_3)$$

$$A = -5.145\,mm \qquad B = -107.567\,mm \qquad D = -109.880\,mm \qquad E = 9.662\,mm$$

$$C := a\cdot\alpha_2\cdot sin(\theta_2) + a\cdot\omega_2^{\,2}\cdot cos(\theta_2) + b\cdot\omega_3^{\,2}\cdot cos(\theta_3) - c\cdot\omega_4^{\,2}\cdot cos(\theta_4)$$

$$C = 2.490\times 10^{4}\,mm\cdot sec^{-2}$$

$$F := a\cdot\alpha_2\cdot cos(\theta_2) - a\cdot\omega_2^{\,2}\cdot sin(\theta_2) - b\cdot\omega_3^{\,2}\cdot sin(\theta_3) + c\cdot\omega_4^{\,2}\cdot sin(\theta_4)$$

$$F = -1.380\times 10^{3}\,mm\cdot sec^{-2}$$

$$\alpha_3 := \frac{C\cdot D - A\cdot F}{A\cdot E - B\cdot D} \qquad \alpha_3 = 231.119\,\frac{rad}{sec^2} \qquad \alpha_4 := \frac{C\cdot E - B\cdot F}{A\cdot E - B\cdot D} \qquad \alpha_4 = -7.768\,\frac{rad}{sec^2}$$

9. Use equation 7.13c to determine the acceleration of point *B* for the crossed circuit.

$$\mathbf{A_B} := c\cdot\alpha_4\cdot\left(-sin(\theta_4) + j\cdot cos(\theta_4)\right) - c\cdot\omega_4^{\,2}\cdot\left(cos(\theta_4) + j\cdot sin(\theta_4)\right)$$

$$\mathbf{A_B} = 8189 + 1239j\,\frac{mm}{sec^2} \qquad A_B := \left|\mathbf{A_B}\right| \qquad \theta_{AB} := arg(\mathbf{A_B}) + \beta$$

The acceleration of pin *B* is $A_B = 8282\,\frac{mm}{sec^2}$ at $\theta_{AB} = -16.4\,deg$ (Global)

PROBLEM 7-28

Statement: The offset slider-crank linkage in Figure P7-8f has the dimensions and crank angle given below. Find $\mathbf{A}_A$, and $\mathbf{A}_B$ in the global coordinate system for the position shown for ω_2 = 25 rad/sec CW, constant. Use an analytical method.

Given: Link lengths:

Link 2 (O_2 to A) $a := 63 \cdot mm$ Link 3 (A to B) $b := 130 \cdot mm$

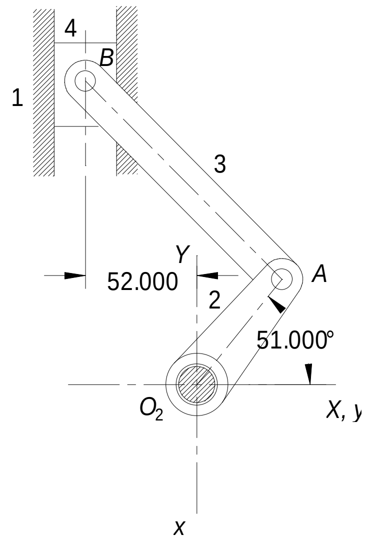
Offset $c := -52 \cdot mm$

Link 2 position, velocity, and acceleration: $\theta_2 := 141 \cdot deg$ $\omega_2 := -25 \cdot \frac{rad}{sec}$ $\alpha_2 := 0 \cdot \frac{rad}{sec^2}$

Coordinate rotation angle: $\alpha := -90 \cdot deg$

Solution: See Figure P7-8f and Mathcad file P0728.

1. Draw the linkage to a convenient scale.



2. Determine θ_3 (in the local coordinate system) and d using equations 4.16 for the crossed circuit.

$$\theta_3 := asin\left(\frac{a \cdot sin(\theta_2) - c}{b}\right) \qquad \theta_3 = 44.828\,deg$$

$$d := a \cdot cos(\theta_2) - b \cdot cos(\theta_3) \qquad d = -141.160\,mm$$

3. Determine the angular velocity of link 3 using equation 6.22a:

$$\omega_3 := \frac{a}{b} \cdot \frac{cos(\theta_2)}{cos(\theta_3)} \cdot \omega_2 \qquad \omega_3 = 13.276\,\frac{rad}{sec}$$

4. Using the Euler identity to expand equation 7.15b for $\mathbf{A}_B$, determine its magnitude, and direction (global).

$$\mathbf{A_A} := a \cdot \alpha_2 \cdot \left(-sin(\theta_2) + j \cdot cos(\theta_2)\right) - a \cdot \omega_2^{\,2} \cdot \left(cos(\theta_2) + j \cdot sin(\theta_2)\right)$$

$$\mathbf{A_A} = 30600.122 - 24779.490j\,\frac{mm}{sec^2} \qquad A_A := \left|\mathbf{A_A}\right| \qquad \theta_{AA} := arg(\mathbf{A_A}) + \alpha$$

The acceleration of pin A is $A_A = 39375\,\frac{mm}{sec^2}$ at $\theta_{AA} = -129.0\,deg$

5. Determine the angular acceleration of link 3 using equation 7.16d.

$$\alpha_3 := \frac{a \cdot \alpha_2 \cdot cos(\theta_2) - a \cdot \omega_2^{\,2} \cdot sin(\theta_2) + b \cdot \omega_3^{\,2} \cdot sin(\theta_3)}{b \cdot cos(\theta_3)} \qquad \alpha_3 = -93.574\,\frac{rad}{sec^2}$$

6. Use equation 7.16e for the acceleration of pin B.

$$A_B := -a \cdot \alpha_2 \cdot sin(\theta_2) - a \cdot \omega_2^{\,2} \cdot cos(\theta_2) + b \cdot \alpha_3 \cdot sin(\theta_3) + b \cdot \omega_3^{\,2} \cdot cos(\theta_3)$$

$A_B = 38274\,\frac{mm}{sec^2}$ A positive sign means that $\mathbf{A}_B$ is downward

PROBLEM 7-31

Statement: The linkage in Figure P7-8d has the dimensions and crank angle given below. Find $\mathbf{A}_A$, $\mathbf{A}_B$, and $\mathbf{A}_{box}$ in the global coordinate system for the position shown for ω_2 = 30 rad/sec CW, constant. Use an analytical method.

Given: Link lengths:

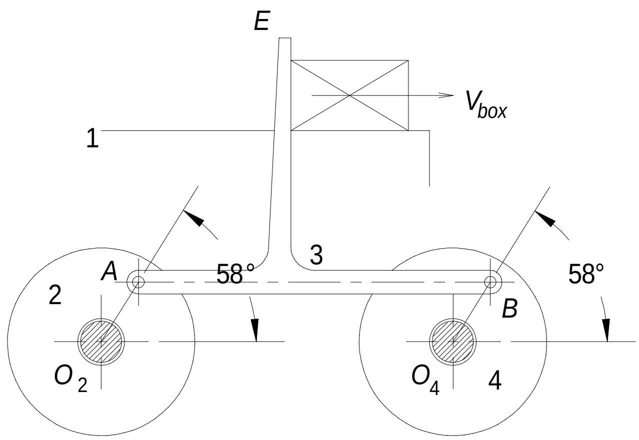
Link 2 (O_2 to A)	$a := 30 \cdot mm$	Link 3 (A to B)	$b := 150 \cdot mm$
Link 4 (O_4 to B)	$c := 30 \cdot mm$	Link 1 (O_2 to O_4)	$d := 150 \cdot mm$

Crank angle: $\theta_2 := 58 \cdot deg$

Input crank angular velocity $\omega_2 := -30 \cdot rad \cdot sec^{-1}$ $\alpha_2 := 0 \cdot rad \cdot sec^{-2}$

Solution: See Figure P7-8d and Mathcad file P0731.

1. Draw the linkage to a convenient scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$K_1 := \frac{d}{a}$ $K_2 := \frac{d}{c}$

$K_1 = 5.0000$ $K_2 = 5.0000$

$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$ $K_3 = 1.0000$

$A := cos(\theta_2) - K_1 - K_2 \cdot cos(\theta_2) + K_3$

$B := -2 \cdot sin(\theta_2)$

$C := K_1 - (K_2 + 1) \cdot cos(\theta_2) + K_3$

$A = -6.1197$ $B = -1.6961$ $C = 2.8205$

3. Use equation 4.10b to find values of θ_4 for the open circuit.

$\theta_4 := 2 \cdot \left(atan2\left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C}\right)\right)$ $\theta_4 = -302.000\ deg$

$\theta_4 := \theta_4 + 360 \cdot deg$ $\theta_4 = 58.000\ deg$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$K_4 := \frac{d}{b}$ $K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b}$ $K_4 = 1.0000$ $K_5 = -5.0000$

$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$ $D = -8.9402$

$E := -2 \cdot \sin(\theta_2)$ $E = -1.6961$

$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$ $F = 0.0000$

5. Use equation 4.13 to find values of θ_3 .

$\theta_3 := 2 \cdot \left(atan2\left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F}\right)\right)$ $\theta_3 = 360.000\,deg$

$\theta_3 := \theta_3 - 360 \cdot deg$ $\theta_3 = 0.000\,deg$

6. Determine the angular velocity of links 3 and 4 using equations 6.18.

$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4 - \theta_2)}{\sin(\theta_3 - \theta_4)}$ $\omega_3 = 0.000 \frac{rad}{sec}$

$\omega_4 := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_4 - \theta_3)}$ $\omega_4 = -30.000 \frac{rad}{sec}$

7. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_{A.}$ Determine the magnitude, and direction.

$\mathbf{A_A} := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^{\,2} \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$

$\mathbf{A_A} = -14308 - 22897i \frac{mm}{sec^2}$ $A_A := |\mathbf{A_A}|$ $\theta_{AA} := arg(\mathbf{A_A})$

The acceleration of pin A is $A_A = 27000 \frac{mm}{sec^2}$ at $\theta_{AA} = -122.0\,deg$

8. Use equations 7.12 to determine the angular accelerations of links 3 and 4.

$A := c \cdot \sin(\theta_4)$ $B := b \cdot \sin(\theta_3)$ $D := c \cdot \cos(\theta_4)$ $E := b \cdot \cos(\theta_3)$

$A = 1.002\,in$ $B = 0.000\,in$ $D = 0.626\,in$ $E = 5.906\,in$

$C := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^{\,2} \cdot \cos(\theta_2) + b \cdot \omega_3^{\,2} \cdot \cos(\theta_3) - c \cdot \omega_4^{\,2} \cdot \cos(\theta_4)$

$C = 0.000\,mm \cdot sec^{-2}$

$F := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^{\,2} \cdot \sin(\theta_2) - b \cdot \omega_3^{\,2} \cdot \sin(\theta_3) + c \cdot \omega_4^{\,2} \cdot \sin(\theta_4)$

$F = 0.000\,mm \cdot sec^{-2}$

$\alpha_3 := \frac{C \cdot D - A \cdot F}{A \cdot E - B \cdot D}$ $\alpha_3 = 0.000 \frac{rad}{sec^2}$ $\alpha_4 := \frac{C \cdot E - B \cdot F}{A \cdot E - B \cdot D}$ $\alpha_4 = 0.000 \frac{rad}{sec^2}$

9. Use equation 7.13c to determine the acceleration of point B.

$$\mathbf{A_B} := c \cdot \alpha_4 \cdot \left(-sin(\theta_4) + j \cdot cos(\theta_4)\right) - c \cdot \omega_4^{\,2} \cdot \left(cos(\theta_4) + j \cdot sin(\theta_4)\right)$$

$$\mathbf{A_B} = -14308 - 22897i \frac{mm}{sec^2} \qquad A_B := \left|\mathbf{A_B}\right| \qquad \theta_{AB} := arg\left(\mathbf{A_B}\right)$$

The acceleration of pin *B* is $A_B = 27000 \frac{mm}{sec^2}$ at $\theta_{AB} = -122.0\,deg$

10. This is a special case Grashof in the parallelogram configuration. All points on link 3 have the same velocity and acceleration. The acceleration of the box will be equal to the X-component of the acceleration of any point on link 3.

$$A_{box} := Re\left(\mathbf{A_A}\right)$$

The acceleration of the box is $A_{box} = -14308 \frac{mm}{sec^2}$ a negative sign means $\mathbf{A}_{box}$ is to the left

PROBLEM 7-34

Statement: The linkage in Figure P7-8g has the dimensions and crank angle given below. Find α_4, $\mathbf{A}_A$, and $\mathbf{A}_B$ in the global coordinate system for the position shown if ω_2 = 15 rad/sec CW and α_2 = 10 rad/sec CCW, constant. Use an analytical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 49 \cdot mm$	Link 3 (A to B)	$b := 100 \cdot mm$
Link 4 (O_4 to B)	$c := 153 \cdot mm$	Link 1 (O_2 to O_4)	$d := 87 \cdot mm$

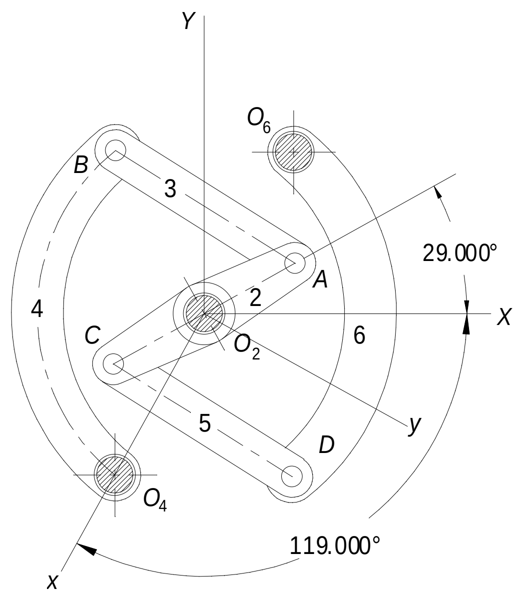
Crank angle: $\theta_2 := 148 \cdot deg$ Local xy system (see layout below)

Input crank angular velocity $\omega_2 := -15 \cdot rad \cdot sec^{-1}$ $\alpha_2 := 10 \cdot rad \cdot sec^{-2}$

Coordinate rotation angle $\beta := -119 \cdot deg$ Global XY system to local xy system

Solution: See Figure P7-8g and Mathcad file P0734.

1. Draw the linkage to scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$K_1 = 1.7755$ $K_2 = 0.5686$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \qquad K_3 = 1.5592$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$A = -0.5821$ $B = -1.0598$ $C = 4.6650$

3. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_4 := 2 \cdot \left(atan2\left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C}\right)\right) \qquad \theta_4 = 208.876\, deg$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \qquad K_4 = 0.8700 \qquad K_5 = 0.3509$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \qquad D = -3.0104$$

$$E := -2 \cdot \sin(\theta_2) \qquad E = -1.0598$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \qquad F = 2.2367$$

5. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3 := 2 \cdot \left(atan2\left(2 \cdot D, -E + \sqrt{E^2 - 4 \cdot D \cdot F}\right)\right) \qquad \theta_3 = 266.892\, deg$$

6. Determine the angular velocity of links 3 and 4 for the crossed circuit using equations 6.18.

$$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4 - \theta_2)}{\sin(\theta_3 - \theta_4)} \qquad \omega_3 = -7.570 \frac{rad}{sec}$$

$$\omega_4 := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_4 - \theta_3)} \qquad \omega_4 = -4.959 \frac{rad}{sec}$$

7. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_{A.}$ Determine the magnitude, and direction (in the local coordinate system).

$$\mathbf{A_A} := a \cdot \alpha_2 \cdot \left(-\sin(\theta_2) + j \cdot \cos(\theta_2)\right) - a \cdot \omega_2^{\,2} \cdot \left(\cos(\theta_2) + j \cdot \sin(\theta_2)\right)$$

$$\mathbf{A_A} = 9090 - 6258j \frac{mm}{sec^2} \qquad A_A := \left|\mathbf{A_A}\right| \qquad \theta_{AA} := arg(\mathbf{A_A}) - \beta$$

The acceleration of pin A is $A_A = 11036 \frac{mm}{sec^2}$ at $\theta_{AA} = 84.46\, deg$

8. Use equations 7.12 to determine the angular accelerations of links 3 and 4 for the crossed circuit.

$$A := c \cdot \sin(\theta_4) \qquad B := b \cdot \sin(\theta_3) \qquad D := c \cdot \cos(\theta_4) \qquad E := b \cdot \cos(\theta_3)$$

$$A = -73.887\, mm \qquad B = -99.853\, mm \qquad D = -133.977\, mm \qquad E = -5.422\, mm$$

$$C := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^{\,2} \cdot \cos(\theta_2) + b \cdot \omega_3^{\,2} \cdot \cos(\theta_3) - c \cdot \omega_4^{\,2} \cdot \cos(\theta_4)$$

$$C = -6.106 \times 10^3\, mm \cdot sec^{-2}$$

$$F := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^{\,2} \cdot \sin(\theta_2) - b \cdot \omega_3^{\,2} \cdot \sin(\theta_3) + c \cdot \omega_4^{\,2} \cdot \sin(\theta_4)$$

$$F = -2.353 \times 10^3\, mm \cdot sec^{-2}$$

$$\alpha_3 := \frac{C \cdot D - A \cdot F}{A \cdot E - B \cdot D} \qquad \alpha_3 = -49.646 \frac{rad}{sec^2} \qquad \alpha_4 := \frac{C \cdot E - B \cdot F}{A \cdot E - B \cdot D} \qquad \alpha_4 = 15.552 \frac{rad}{sec^2}$$

9. Use equation 7.13c to determine the acceleration of point B for the crossed circuit.

$$\mathbf{A_B} := c \cdot \alpha_4 \cdot \left(-\sin(\theta_4) + j \cdot \cos(\theta_4)\right) - c \cdot \omega_4^{\,2} \cdot \left(\cos(\theta_4) + j \cdot \sin(\theta_4)\right)$$

$$\mathbf{A_B} = 4444 - 267j \frac{mm}{sec^2} \qquad A_B := \left|\mathbf{A_B}\right| \qquad \theta_{AB} := arg(\mathbf{A_B}) - \beta$$

The acceleration of pin B is $A_B = 4452 \frac{mm}{sec^2}$ at $\theta_{AB} = 115.6\, deg$

PROBLEM 7-41

Statement: Figure P7-11 shows a linkage that operates at 500 crank rpm. Find and plot the magnitude and direction of the acceleration of point B at 2-deg increments of crank angle. Check your result with program FOURBAR.

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Link lengths:

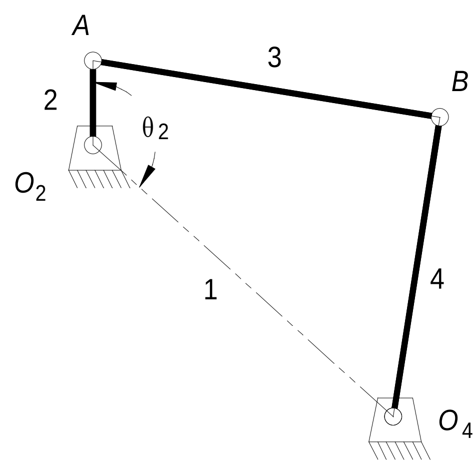
Link 2 (O_2 to A)	$a := 2.000 \cdot in$	Link 3 (A to B)	$b := 8.375 \cdot in$
Link 4 (B to O_4)	$c := 7.187 \cdot in$	Link 1 (O_2 to O_4)	$d := 9.625 \cdot in$

Input crank angular velocity $\omega_2 := 500 \cdot rpm$ $\omega_2 = 52.360\, rad \cdot sec^{-1}$

$\alpha_2 := 0 \cdot rad \cdot sec^{-2}$

Solution: See Figure P7-11 and Mathcad file P0741.

1. Draw the linkage to scale and label it.



2. Determine the range of motion for this Grashof crank rocker.

$$\theta_2 := 0 \cdot deg, 2 \cdot deg .. 360 \cdot deg$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 4.8125 \qquad K_2 = 1.3392$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \qquad K_3 = 2.7186$$

$$A(\theta_2) := cos(\theta_2) - K_1 - K_2 \cdot cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot sin(\theta_2) \qquad C(\theta_2) := K_1 - (K_2 + 1) \cdot cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_4(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \qquad K_4 = 1.1493 \qquad K_5 = -3.4367$$

$$D(\theta_2) := cos(\theta_2) - K_1 + K_4 \cdot cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

7. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

$$\omega_3(\theta_2) := \frac{a\cdot\omega_2}{b}\cdot\frac{\sin(\theta_4(\theta_2) - \theta_2)}{\sin(\theta_3(\theta_2) - \theta_4(\theta_2))}$$

$$\omega_4(\theta_2) := \frac{a\cdot\omega_2}{c}\cdot\frac{\sin(\theta_2 - \theta_3(\theta_2))}{\sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

8. Use equations 7.12 to determine the angular acceleration of link 4.

$$A(\theta_2) := c\cdot\sin(\theta_4(\theta_2)) \qquad B(\theta_2) := b\cdot\sin(\theta_3(\theta_2))$$

$$D(\theta_2) := c\cdot\cos(\theta_4(\theta_2)) \qquad E(\theta_2) := b\cdot\cos(\theta_3(\theta_2))$$

$$C(\theta_2) := a\cdot\alpha_2\cdot\sin(\theta_2) + a\cdot\omega_2^{\,2}\cdot\cos(\theta_2) + b\cdot\omega_3(\theta_2)^2\cdot\cos(\theta_3(\theta_2)) - c\cdot\omega_4(\theta_2)^2\cdot\cos(\theta_4(\theta_2))$$

$$F(\theta_2) := a\cdot\alpha_2\cdot\cos(\theta_2) - a\cdot\omega_2^{\,2}\cdot\sin(\theta_2) - b\cdot\omega_3(\theta_2)^2\cdot\sin(\theta_3(\theta_2)) + c\cdot\omega_4(\theta_2)^2\cdot\sin(\theta_4(\theta_2))$$

$$\alpha_4(\theta_2) := \frac{C(\theta_2)\cdot E(\theta_2) - B(\theta_2)\cdot F(\theta_2)}{A(\theta_2)\cdot E(\theta_2) - B(\theta_2)\cdot D(\theta_2)}$$

9. Determine the acceleration of point *B* using equations 7.13c.

$$\mathbf{A_B}(\theta_2) := c\cdot\alpha_4(\theta_2)\cdot(-\sin(\theta_4(\theta_2)) + j\cdot\cos(\theta_4(\theta_2))) - c\cdot\omega_4(\theta_2)^2\cdot(\cos(\theta_4(\theta_2)) + j\cdot\sin(\theta_4(\theta_2)))$$

$$A_B(\theta_2) := |\mathbf{A_B}(\theta_2)| \qquad \theta_{AB}(\theta_2) := \arg(\mathbf{A_B}(\theta_2))$$

10. Plot the magnitude and angle of the acceleration at point *B*.